

Mesoscopic Models of Turbulence, Superfluidity and Superconductivity

Alexander Kaivarainen

Department of Applied Mathematics and Cybernetics, University of Petrozavodsk,
185000, Petrozavodsk, Russia

Home Page: <http://kftt.karelia.ru/~alexk>
alexk@kftt.karelia.ru

Materials, presented in this original article are based on following publications:

- [1]. A. Kaivarainen. Book: Hierarchic Concept of Matter and Field. Water, biosystems and elementary particles. New York, NY, 1995 and 2nd edition of this book. (see URL: <http://kftt.karelia.ru/~alexk>).
- [2]. A. Kaivarainen. New Hierarchic Theory of Matter General for Liquids and Solids: dynamics, thermodynamics and mesoscopic structure of water and ice (see URL: <http://kftt.karelia.ru/~alexk>).
- [3]. A. Kaivarainen. Hierarchic Concept of Condensed Matter : Interrelation between mesoscopic and macroscopic properties (see URL: <http://kftt.karelia.ru/~alexk>).

Contents of Article

Summary of Part I of book: "Hierarchic concept of matter"

- 1 Turbulence. General description
- 2 Mesoscopic mechanism of turbulence
- 3 Superfluidity. General description
- 4 Mesoscopic scenario of fluidity
- 5 Superfluidity as a hierarchic self-organization process
- 6 Superfluidity in ^3He

- 7 Superconductivity
General properties of metals and semiconductors
Plasma oscillations
Cyclotron resonance
Electroconductivity
8. Microscopic theory of superconductivity (BCS)
9. Mesoscopic scenario of superconductivity
Interpretation of experimental data in the framework of mesoscopic model of superconductivity

To better understanding of this article the reading of papers [2, 3] is recommended.

Summary to Part I of book:
Hierarchic Theory of Matter and Field (by: A. Kaivarainen)

A basically new hierarchic quantitative theory, general for solids and liquids, has been developed.

It is assumed, that anharmonic oscillations of particles in any condensed matter lead to emergence of three-dimensional (3D) superposition of standing de Broglie waves of molecules, electromagnetic and acoustic waves. Consequently, any condensed matter could be considered as a gas of 3D standing waves of corresponding nature. Our approach unifies and develops strongly the Einstein's and Debye's models.

Collective excitations, like 3D standing de Broglie waves of molecules, representing at certain conditions the molecular Bose condensate, were analyzed, as a background of hierarchic model of condensed matter.

The most probable de Broglie wave (wave B) length is determined by the ratio of Plank constant to the most probable impulse of molecules, or by ratio of its most probable phase velocity to frequency. The waves B are related to molecular translations (tr) and librations (lb).

As the quantum dynamics of condensed matter does not follow in general case the classical Maxwell-Boltzmann distribution, the real most probable de Broglie wave length can exceed the classical thermal de Broglie wave length and the distance between centers of molecules many times.

This makes possible the atomic and molecular Bose condensation in solids and liquids at temperatures, below boiling point. It is one of the most important results of new theory, which we have confirmed by computer simulations on examples of water and ice.

Four strongly interrelated new types of quasiparticles (collective excitations) were introduced in our hierarchic model:

1. *Effectons (tr and lb)*, existing in "acoustic" (a) and "optic" (b) states represent the coherent clusters in general case;
2. *Convertons*, corresponding to interconversions between *tr* and *lb* types of the effectons (flickering clusters);
3. *Transitons* are the intermediate [$a \rightleftharpoons b$] transition states of the *tr* and *lb* effectons;
4. *Deformons* are the 3D superposition of IR electromagnetic or acoustic waves, activated by *transitons* and *convertons*.

Primary effectons (tr and lb) are formed by 3D superposition of the **most probable standing de Broglie waves** of the oscillating ions, atoms or molecules. The volume of effectons (tr and lb) may contain from less than one, to tens and even thousands of molecules. The first condition means validity of **classical** approximation in description of the subsystems of the effectons. The second one points to **quantum properties of coherent clusters due to molecular Bose condensation**.

The liquids are semiclassical systems because their primary (tr) effectons contain less than one molecule and primary (lb) effectons - more than one molecule. *The solids are quantum systems totally because both kind of their primary effectons (tr and lb) are molecular Bose condensates.* **These consequences of our theory are confirmed by computer calculations.**

The 1st order [*gas* → *liquid*] transition is accompanied by strong decreasing of rotational (librational) degrees of freedom due to emergence of primary (lb) effectons and [*liquid* → *solid*] transition - by decreasing of translational degrees of freedom due to Bose-condensation of primary

(tr) effectons.

In the general case the effecton can be approximated by parallelepiped with edges corresponding to de Broglie waves length in three selected directions (1, 2, 3), related to the symmetry of the molecular dynamics. In the case of isotropic molecular motion the effectons' shape may be approximated by cube.

The edge-length of primary effectons (tr and lb) can be considered as the "parameter of order".

The in-phase oscillations of molecules in the effectons correspond to the effecton's (a) - *acoustic* state and the counterphase oscillations correspond to their (b) - *optic* state. States (a) and (b) of the effectons differ in potential energy only, however, their kinetic energies, impulses and spatial dimensions - are the same. The *b*-state of the effectons has a common feature with **Frölich's polar mode**.

The ($a \rightarrow b$) or ($b \rightarrow a$) transition states of the primary effectons (tr and lb), defined as primary transistons, are accompanied by a change in molecule polarizability and dipole moment without density fluctuations. At this case they lead to absorption or radiation of IR photons, respectively.

Superposition (interception) of three internal standing IR photons of different directions (1,2,3) - forms primary electromagnetic deformons (tr and lb).

On the other hand, the [$lb \rightleftharpoons tr$] *convertions* and *secondary transistons* are accompanied by the density fluctuations, leading to *absorption or radiation of phonons*.

Superposition resulting from interception of standing phonons in three directions (1,2,3), forms **secondary acoustic deformons (tr and lb)**.

Correlated collective excitations of primary and secondary effectons and deformons (tr and lb), localized in the volume of primary *tr* and *lb electromagnetic* deformons, lead to origination of **macroeffectons, macrotransistons and macrodeformons** (tr and lb respectively).

Correlated simultaneous excitations of tr and lb macroeffectons in the volume of superimposed *tr* and *lb electromagnetic* deformons lead to origination of **supereffectons**.

In turn, the coherent excitation of *both: tr and lb macrodeformons and macroconvertions* in the same volume means creation of **superdeformons**. Superdeformons are the biggest (cavitational) fluctuations, leading to microbubbles in liquids and to local defects in solids.

Total number of quasiparticles of condensed matter equal to $4! = 24$, reflects all of possible combinations of the four basic ones [1-4], introduced above. This set of collective excitations in the form of "gas" of 3D standing waves of three types: de Broglie, acoustic and electromagnetic - is shown to be able to explain virtually all the properties of all condensed matter.

The important positive feature of our hierarchic model of matter is that it does not need the semi-empiric intermolecular potentials for calculations, which are unavoidable in existing theories of many body systems. The potential energy of intermolecular interaction is involved indirectly in dimensions and stability of quasiparticles, introduced in our model.

The main formulae of theory are the same for liquids and solids and include following experimental parameters, which take into account their different properties:

[1]- **Positions of (tr) and (lb) bands in oscillatory spectra;**

[2]- **Sound velocity;**

[3]- **Density;**

[4]- **Refraction index (extrapolated to the infinitive wave length of photon).**

The knowledge of these four basic parameters at the same temperature and pressure makes

it possible using our computer program, to evaluate more than 150 important characteristics of any condensed matter. Among them are such as: total internal energy, kinetic and potential energies, heat-capacity and thermal conductivity, surface tension, vapor pressure, viscosity, coefficient of self-diffusion, osmotic pressure, solvent activity, etc. Most of calculated parameters are hidden, i.e. inaccessible to direct experimental measurement.

The new interpretation and evaluation of Brillouin light scattering and Mössbauer effect parameters may also be done on the basis of hierarchic theory. Mesoscopic scenarios of turbulence, superconductivity and superfluidity are elaborated.

Some original aspects of water in organization and large-scale dynamics of biosystems - such as proteins, DNA, microtubules, membranes and regulative role of water in cytoplasm, cancer development, quantum neurodynamics, etc. have been analyzed in the framework of Hierarchic theory.

Computerized verification of our Hierarchic concept of matter on examples of water and ice is performed, using special computer program: Comprehensive Analyzer of Matter Properties (CAMP, copyright, 1997, Kaivarainen). The new opto-acoustical device (CAMP), based on this program, with possibilities much wider, than that of IR, Raman and Brillouin spectrometers, has been proposed (see URL: <http://www.karelia.ru/~alexk>).

This is the first theory able to predict all known experimental temperature anomalies for water and ice. The conformity between theory and experiment is very good even without any adjustable parameters.

The hierarchic concept creates a bridge between micro- and macro- phenomena, dynamics and thermodynamics, liquids and solids in terms of quantum physics.

1. Turbulence. General description

The type of flow when particles move along the straight trajectory without mixing with adjoining layers, is termed laminar flow.

If the layers of the liquid of the laminar flow are moving relative to each other at different velocities, then the forces of internal friction (F_{fr}) or viscosity forces originate between them:

$$F_{fr} = \eta \left| \frac{\Delta v}{\Delta d} \right| S, \quad 1$$

where: Δv is relative liquid layer velocity; S is the contact surface; η is dynamic viscosity; $\left| \frac{\Delta v}{\Delta d} \right|$ is the module of velocity gradient directed to the surface of layers.

Near the walls of a straight tube the velocity of laminar flow is equal to zero and in the center of the tube it is maximum.

The relation between the layer velocity and its distance from central axes of the tube (r) is parabolic:

$$v(r) = v_0 \left(1 - \frac{r^2}{a_t^2} \right) \quad 2$$

where: a_t is tube radius; v_0 is the velocity of the liquid on the central axis, depending on the difference of pressure at the ends of the tube:

$$\Delta P = P_1 - P_2 \quad 3$$

as follows:

$$v_0 = \frac{P_1 - P_2}{4\eta l} a_t^2 \quad 4$$

where: l is tube length and η is dynamic viscosity.

The flux of liquid, i.e. the volume of liquid flowing over the cross-section of the tube during a time unit is determined by the Poiseuille formula:

$$Q = \frac{(P_1 - P_2)\pi a_t^2}{8\eta l} \quad 5$$

This formula has been used frequently for estimation of dynamic viscosity η .

The corresponding mass of flowing liquid is equal:

$$m = \rho Q \quad 6$$

and corresponding kinetic energy:

$$T_k = \frac{\rho}{4} Q v_0^2 \quad 7$$

where: ρ is density of liquid.

The work of internal friction force is:

$$A = -4\eta v_0 l Q / \rho R^2$$

In the case of the laminar movement of a spherical body relative to liquid the force of internal friction (viscosity force) is determined by the Stokes law:

$$F_{fr} = 6\pi a v \eta, \quad 8$$

where: (a) is the radius of sphere and v is its relative velocity.

As a result of liquid velocity (v) and/or the characteristic dimension (a) increasing, the laminar type of liquid flow could change to the turbulent one.

This begins at certain values of the dimensionless Reynolds number:

$$R = \rho v_c a / \eta = v_c a / \nu, \quad 9$$

where: ρ is liquid density; v_c is characteristic (average) flow velocity; $\nu = \eta / \rho$ is **kinematic viscosity** of liquid.

For a round tube with radius (a) the critical value of R is about 1000. A turbulent type of flow

is accompanied by rapid irregular pulsations of liquid velocity and pressure, representing a kind of self-organization.

In the case of nonstationary movement, the flow can be characterized by two additional dimensionless parameters like:

Strouhal number:

$$S = v_c \tau / a \quad 10$$

where: τ is the characteristic time of velocity (v_c) pulsations;
and *March number:*

$$M = v_c / v_s \quad 11$$

where: v_s is sound velocity in liquid.

2. Mesoscopic mechanism of turbulence

The physical scenario of transition from a laminar flow to a turbulent one is still unclear. It is possible to propose the mechanism of this transition based on mesoscopic concept of matter.

Let us start from the assumption that in the case of laminar flow, the thickness of parallel layers is determined by the thickness of primary electromagnetic deformons (translational $\sim 5 \cdot 10^5 \text{ \AA}$ and librational $\sim 10^5 \text{ \AA}$), equal to linear dimensions of corresponding macrodeformons.

The total internal energy and the internal pressure of neighboring layers are not equal.

The surface between such layers can be characterized by corresponding surface tension $[\sigma(r)]$. Surface tension prevents mixing between layers with different laminar flow velocities.

According to our model the thickness of the two outer borders of each layer (skin-surface) is determined by the effective linear dimensions of primary (tr and lib) effectons $[l_{tr,lb} \sim (3 - 15) \text{ \AA}]$ related to the corresponding most probable wave B length ($\lambda_{tr,lb}$) of molecules in liquid (see eq. 11.30):

$$l_{tr,lb} = (V_{ef}/V_{ef}^{2/3}) \quad 12$$

Decreasing of $l_{tr,lb}$, depending on most probable impulse of liquid molecules, as a result of increased flow velocity $[v^1(r)]$ see eq. (13) and/or temperature elevation in accordance with our theory of surface tension (see eq. 11.33 of [1] or [3]), leads to reducing of $\sigma(r)$. In turn, this effect strongly decreases the work of cavitation fluctuations, i.e. the bubbles formation (27) and increases their concentration (29). **These bubbles lead to mixing of laminar layers, the instability of laminar flow and its changing to a turbulent type.**

The critical flow velocity: $v_c = v^1(r)$ is determined by the critical librational wave B length:

$$\lambda_{lb}^{1,2,3} = h/m \cdot \left[\left(v_{gr}^{1,2,3} \right)_{lb} + v^1(r) \right] \quad 13$$

corresponding to the condition (6.6) of liquid-gas first order phase transition:

$$[(V_{ef})_{lib}/(V_0/N_0)] = \left[\frac{9}{4\pi} (\lambda^{(1)} \cdot \lambda^{(2)} \cdot \lambda^{(3)})_{lib}/(V_0/N_0) \right] \leq 1 \quad 14$$

where: m is molecular mass; $v_{gr}^{1,2,3}$ is the most probable librational group velocity of molecules of liquid in selected directions (1, 2, 3); $v^1(r)$ is the flow velocity of a liquid layer in the tube at the distance (r) from the central axes of the tube (2).

Increasing of $v^1(r)$ at $r \rightarrow 0$ decreases λ_{lb}^1 and $(V_{ef})_{lb}$ in accordance with (13) and (14). The value of λ_{lb}^1 is also related to phase velocity (v_{ph}^a) and frequency (ν_1^a) of the primary librational effecton in (a) state (see 2.60):

$$\lambda_{lib}^1 = h/m \left[(v_{gr}^1)_{lb} + v^1(r) \right] = \left(\frac{v_{ph}^a}{\nu_1^a} \right)_{lb} =$$

$$= (v_{ph}^a/v_p^1) \left[\exp\left(h(v_p^1)_{lb}/kT - 1 \right) \right] \quad 15$$

where:

$$(v_p^1)_{lb} = (v_1^b - v_1^a)_{lb} = c(v_p^1)_{lb} \quad 16$$

is the frequency of $(a \leftrightarrow b)_{lb}$ transitions of the primary librational effecton of a **flowing** liquid determined by librational band wave number $(\tilde{\nu}_p^1)_{lb}$ in the oscillatory spectra.

It was calculated earlier for water under stationary conditions, that the elevation of temperature from 0^0 to 100^0C till the phase transition condition (14), is accompanied by the increase in $(v_{gr})_{lb}$ from $(1.1 \text{ to } 4.6) \cdot 10^3 \text{ cm/s}$ (Fig. 12b of [1]).

This means that at 30^0C , when $(v_{gr})_{lb} \simeq 2 \cdot 10^3 \text{ cm/s}$, the critical flow velocity $v^1(r)$, necessary for mechanical boiling of water (condition 14) should be about 2.5 m/s .

The reduced number of primary librational effectons (N_{ef}) in the volume (V_D^M) of primary electromagnetic deformons (tr and lib) also increases with temperature and/or flow velocity:

$$\left(N_{ef} \right)_{tr,lb}^D = \left[\frac{P_a + P_b}{Z} n_{ef} \cdot V_D^M \right] \quad 17$$

The reduced number of primary transits (N_t) has a similar dependence on T and $v^1(r)$, due to increasing of n_{ef} , and V_D^M as far:

$$\left(N_t \right)_{tr,lb}^D = \left(N_{ef} \right)_{tr,lb}^D \quad 18$$

The analysis of eq.(15) *predicts* that at $T = const$ an increase in $v^1(r)$ must be accompanied by the low-frequency shift of the librational band: $\tilde{\nu}_{lib}^{(1)} \simeq 700 \text{ cm}^{-1}$ and/or by the decrease in sound velocity $(v_{ph}^a)_{lb}$ (eq.2.74 of [1]) in the direction of flow.

It follows also from our mesoscopic theory that these changes should be accompanied by a rise in dynamic viscosity (11.45 of [1] or [3]) due to increased structural factor (T_{kin}/U_{tot}) .

Turbulent pulsations of flow velocity (Δv) originate under developed turbulence conditions:

$$\Delta v = v_{tur} = v - \bar{v} \quad 19$$

where: \bar{v} is average flow velocity and (v) is instant flow velocity.

The frequencies of large-scale pulsations have the order of:

$$v = \bar{v}/\lambda, \quad 20$$

where: λ is the main scale of pulsations.

λ can correlate with the dimensions of electromagnetic deformons and can be determined by the transverse convection rate, depending on the bubbles dimensions.

The pulsations of flow velocity (Δv) can result from the

a) mixing of parallel layers with different flow velocity and

b) fluctuation of viscosity force (eq. 8) due to fluctuations in bubbles radius and concentration, as well as density, viscosity and thermal conductivity;

c) movements and emergence of the bubbles as a result of the Archimed force.

The bubbles have two opposite types of influence on *instant* velocity. The layer mixing effect induced by them can increase flow velocity.

On the other hand, the bubbles can simultaneously decrease flow velocity due to enhanced internal friction.

In the case of developed turbulence with different scales of pulsations it is reasonable to introduce the characteristic Reynolds number:

$$R_\lambda = v_{tur}\lambda/v_{tur} \quad 21$$

where: λ is a scale of pulsations; v_{tur} is the velocity of pulsation and $v_{tur} = (\eta/\rho)_{tur}$ characteristic kinematic viscosity.

The ratio between turbulent kinematic viscosity (v_{tur}) and a laminar one (v) is related to the corresponding Reynolds numbers like (Landau, Lifshits, 1988):

$$\frac{v_{tur}}{V} \sim \frac{R}{R_{tur}} \quad 22$$

One can see from (21) that it means:

$$v_{tur} \cdot \lambda \sim const \quad 23$$

Based on dimension relations turbulent kinematic viscosity can be expressed as follows:

$$v_{tur} \sim \Delta v \cdot l \sim v_\lambda \cdot \lambda \quad 24$$

and energy dissipation as:

$$\epsilon_{dis} \sim v_{tur}(v_{tur}/\lambda)^2 \sim \frac{v_{tur}^3}{\lambda} \quad 25$$

This expression leads to the Kholmogorov-Obuchov law:

$$v_{tur} \sim (\epsilon_{dis}\lambda)^{1/3} \quad 26$$

Large-scale pulsations correspond to high λ values and low v_{tur} values. i.e. high characteristic turbulence Reynolds numbers (see 21).

According to our model the maximum energy dissipation occurs in the volume of superdeformons (or supertransitons).

Mechanically induced boiling under conditions of turbulence (eqs. 13 and 14) is accompanied by the emergence of gas bubbles related to the increased superdeformons probability and decreased surface tension between layers.

Critical bubble creation work (W) is strongly dependent on inter- layer surface tension (σ). A general classical theory (Nesis, 1973) gives:

$$W = \frac{4}{3}\pi a^2 \sigma = \frac{16\pi\sigma^3}{3(P - P_{ext})^2} \quad 27$$

where:

$$P = P_{ext} + \frac{2\sigma}{a} = P_{a=\infty} \cdot \exp\left(-\frac{2\sigma V_e}{akT}\right) \quad 28$$

P is the internal gas pressure in a bubble with radius (a); V_e is the volume of liquid occupied by one molecule.

The bubbles quantity (N_b) has an exponential dependence on W :

$$N_b = \exp\left(-\frac{W}{kT}\right) \quad 29$$

One can see from our mesoscopic theory of surface tension (eqs. 11.31 - 11.33 of [1]; [3] and eq.12) that under **mechanical boiling conditions** the skin-surface thickness (12) : $l \rightarrow (V_0/N_0)^{1/3}$ and $q^s \rightarrow 1$, the interlayer surface tension (σ) tends to zero, W decreases and N_b increases.

We can conclude that the mesoscopic scenario of mechanical boiling presented here can provide a background for elaboration of a quantitative physical theory of turbulence and other hydrodynamic instabilities like Taylor's and Benar's ones.

3. Superfluidity. General description

Superfluidity has been revealed for two liquids only: helium isotopes: ^4He with boson's properties ($S = 0$) and ^3He with fermion properties ($S = 1/2$). The interactions between the atoms of these liquids is very weak. It will be shown below that the values of normal sound velocity at temperatures higher than those of second order phase transition (λ -point) are lower than the most probable thermal velocities of the atoms of these liquids.

The first theories of superfluidity were proposed by Landau (1941) and Feynman (1953).

First order phase transition [gas \rightarrow liquid] occurs at 4.22K.

Second order phase transition, when superfluidity originates, $^4\text{He} \rightarrow \text{He II}$ takes place at $T_\lambda = 2.17\text{K}$ ($P_{\text{ext}} = 1 \text{ atm}$). This transition is accompanied by:

- a) heat capacity jump to higher values;**
- b) abruptly increased thermal conductivity;**
- c) markedly decreased cavitation fluctuations and bubbles in liquid helium.**

For explanation of experimental data Landau supposed that at $T < T_\lambda$ the He II consists of two components:

- the *superfluidity component* with relative fraction of density ρ_s/ρ , increasing from zero at $T = T_\lambda$ to 1 at $T = 0 \text{ K}$. The properties of this component are close to those of an ideal liquid with a potential type of flow. The entropy of this component is zero and it does not manifest the viscous friction on flowing through narrow capillaries;
- the *normal component* with density

$$\rho_n = \rho - \rho_s \quad 30$$

decreasing from 1 at $T = T_\lambda$ to zero at $T = 0 \text{ K}$. This component behaves as a usual viscous liquid which exhibits dumping of the oscillating disk in He II. Landau considered this component to be a gas of two types of excitations: *phonons* and *rotons*.

The hydrodynamics of normal and superfluid components of He II are characterized by *two velocities*: normal (v_n) and superfluid one:

$$v_{sf} = (\hbar/m)\nabla\varphi \quad 31$$

where $\nabla\varphi \sim k_{sf} = 1/L_{sf}$ is a phase of Bose-condensate wave function - see eq. 36.

As a result of two types of hydrodynamic velocities and densities, the corresponding 2 types of sound waves propagate **in the volume of He II**.

The *first sound* (U_1) is determined by the usual formula valid for normal condensed matter:

$$U_1^2 = (\partial P/\partial \rho)_s \quad 32$$

In this case density oscillations spread in the form of phonons.

The *second sound* (U_2) is related to oscillations of temperature and entropy (S):

$$U_2^2 = \rho_s T S^2 / c \rho_n \quad 33$$

In normal condensed media the temperature oscillation fade at the distance of the order of wave length.

Landau considered the second sound as density waves in the gas of quasiparticles: **rotons and phonons**.

The third sound (U_3) propagates in the thin surface films of He II in the form of "ripples", i.e. quantum capillary waves related to the isothermal oscillations of the superfluid component.

$$U_3 = (\rho_s/\rho_s) \cdot d \cdot \frac{\partial E}{\partial d} (1 + TS/L), \quad 34$$

where: $(\bar{\rho}_s/\rho_s)$ is the relative density of superfluid component averaged in the thickness of the film (d); E is the potential of Van- der-Waals interactions of ^4He atoms with the bottom surface; L is evaporation heat.

The *fourth sound* (U_4) propagates in He II, located in very narrow capillaries, when the length of quasiparticles (phonons and rotons) free run is compatible or bigger than the diameter of these capillaries or pores.

The hydrodynamic velocity (v_n) of the normal component under such conditions is zero and $\rho_n/\rho \ll \rho_{sf}/\rho$:

$$U_4^2 = (\rho_s/\rho)U_1^2 + (\rho_n/\rho)U_2^2 \simeq (\rho_s/\rho)U_1^2 \quad 35$$

In accordance with Bose-Einstein statistics, a decrease in temperature, when $T \rightarrow T_\lambda$, leads to condensation of bosons in a minimum energy state.

This process results in the origination of a superfluid component of He II with the coherent thermal and hydrodynamic movement of atoms.

Coherence means that this movement can be described by the single wave function:

$$\psi = \rho_S^{1/2} \cdot e^{i\varphi} \quad 36$$

The movement of the superfluid component is *potential* as far its velocity (\vec{v}_{sf}) is determined by eq.31 and:

$$\text{rot } v_{sf} = 0 \quad 37$$

Vortex filaments in He II

When the rotation velocity of a cylindrical vessel containing He II is high enough, then the emergency of so-called vortex filaments becomes thermodynamically favorable. The filament is formed by the superfluid component of He II in such a way that their impulse of movement decreases the total energy of He II in a rotating vessel.

The shape of filaments in this case is like a straight rod and their *thickness* is of the order of atom's dimensions, increasing with lowering the temperature at $T < T_\lambda$.

Vortex filaments are continuous. They are closed or limited within the boundaries of a liquid. For each surface surrounding a vortex filament the condition (37) is valid.

The values of velocity of circulation around the axis of filaments are determined (Landau, 1941) as follows:

$$\oint v_{sf} dl = 2\pi r v_{sf} = 2\pi\kappa \quad 38$$

and

$$v_{sf} = \kappa/r \quad 39$$

Increasing the radius of circulation (r) leads to decreased circulation velocity (v_{sf}). Substituting v_{sf} in eq.31, we obtain:

$$\oint v_{sf} dl = \frac{\hbar}{m} \Delta\Phi, \quad 40$$

where: $\Delta\Phi = n2\pi$ is a phase change as a result of circulation, $n = 1, 2, 3, \dots$ is the integer number.

Comparing (40) and (38) gives:

$$\kappa = n \frac{\hbar}{m} \quad 41$$

It has been shown that only curls with $n = 1$ are thermodynamically stable.

Taking this into account, we have from (39) and (41):

$$r = n \frac{\hbar}{m v_{sf}} \quad 42$$

An increase in the angle frequency of rotation of the cylinder containing He II results in the increased density distribution of vortex filaments on the cross-section of the cylinder.

As a result of interaction between the filament and the normal component of He II, the filaments move in the rotating cylinder with normal liquid.

The flow of He II through the capillaries can be accompanied by emergence of vortex filaments.

In ring-shaped vessels the circulation of closed vortex filaments is stable. Stability is related to the quantum pattern of circulation change (eqs. 38 and 41).

Let us consider now the phenomena of superfluidity in He II in the framework of our mesoscopic concept.

4. Mesoscopic scenario of fluidity

It will be shown below how our mesoscopic model (Table 1) can be used to explain He II properties, its excitation spectrum (Fig. 1), increased heat capacity at λ -point and the vortex filaments formation.

We assume here, that the formulae obtained earlier for internal energy (U_{tot} – eq.4.3), viscosity (eqs. 11.48, 11.49 and 11.55 of [1] or see [3]), thermal conductivity (eq. 11.37), vapor pressure (eq. 11.26) remain valid for both components of He II.

The theory proposed by Landau (Lifshits, Pitaevsky, 1978) qualitatively explains only the lower branch (a) in the spectrum (Fig. 1) as a result of phonons and rotons excitation.

But the upper branch (b) points that the real process is more complicated and needs introduction of other quasiparticles and excited states for its explanation.

Our mesoscopic model of superfluidity interrelates the lower branch with the ground acoustic (a) state of primary effectons in liquid ^4He and the upper branch with their excited optical (b) state. In accordance with our model, the dissipation and viscosity friction (see section 11.6) arise in the normal component of He II due to thermal phonons radiated and absorbed in the course of the $\bar{b} \rightarrow$ and $\rightarrow \bar{b}$ transitions of secondary effectons correspondingly, symbatic to macrodeformons excitation.

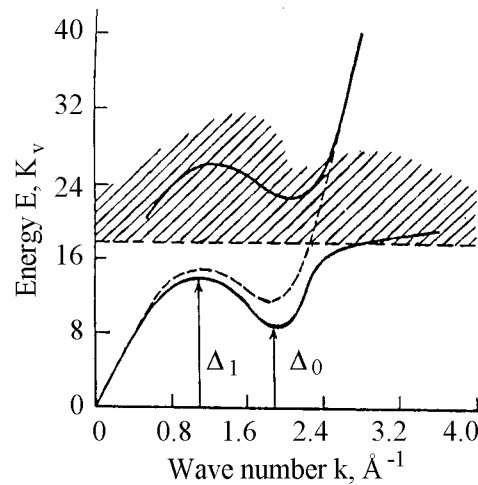


Fig. 1. Excitation spectrum of liquid ^4He from neutron scattering measurements (March and Parrinello, 1982). Spectrum is characterized by two branches, corresponding to (a) and (b) states of the primary effectons according to the mesoscopic model.

Landau described the minimum in the region of λ -point using the expression:

$$E = \Delta_0 + \frac{(P - P_0)^2}{2m^*}, \quad 43$$

where Δ_0 and P_0 are the energy and impulse of liquid ^4He at λ -point (Fig. 1) and $m^* = 0.16m$ is the effective mass of the ^4He atom ($m_{\text{He}} = 4 \cdot 1.44 \cdot 10^{-24} \text{g} = 5.76 \cdot 10^{-24} \text{g}$). The effective mass m^* can be determined experimentally.

Feynman (1953) explained the same part of the excitation spectra by the nonmonotonic behavior of the structure factor $S(k)$ and the formula:

$$E = \hbar\omega = \frac{\hbar^2 k^2}{2mS} = \frac{\hbar^2}{2mL^2 S} \quad 44$$

where:

$$k = 1/L = 2\pi/\lambda \quad 45$$

is the wave number of neutron interacting with liquid ^4He .

Our mesoscopic model allows to unify Landau's and Feynman's approaches.

The total energy of de Broglie wave either free or as part of condensed matter can be expressed through its amplitude squared (A^2), or effective mass (m^*) in the following manner (see 2.45 and 2.46):

$$E_{\text{tot}} = T_k + V = mv_{gr}v_{ph} = \frac{\hbar^2}{2mA^2} = \frac{\hbar^2}{2m^*L^2} \quad 46$$

where v_{gr} and v_{ph} are the most probable group and phase velocities.

In accordance with our model (eq. 2.46a), the structural factor $S(k)$ is equal to the kinetic (T_k) to total (E_{tot}) energy ratio of wave B:

$$S = T_k/E_{\text{tot}} = A^2/L^2 = m^*/m \quad 47$$

where:

$$T_k = P^2/2m = \frac{\hbar^2}{2mL} \quad 48$$

Combining (46), (47) and (48), we obtain the following set of equation for the energy of ${}^4\text{He}$ at transition λ -point:

$$\left. \begin{aligned} \Delta_0 = E_0 &= \frac{\hbar^2}{2mA_0^2} = \frac{\hbar^2}{2m^*L_0^2} \\ \Delta_0 &= \frac{\hbar^2}{2mL_0^2S} = \frac{T_k^0}{S} \end{aligned} \right\} \quad 49$$

These approximate formulae for the total energy of liquid ${}^4\text{He}$ made it possible to estimate the most probable wave B length, forming the primary librational (or rotational effectons) at λ -point:

$$\lambda_0 = \frac{\hbar}{mv_{gr}^0} = 2\pi L_0 = 2\pi A_0 \left(m/m^* \right)^{1/2}, \quad 50$$

where the critical amplitude of wave B:

$$A_0 = \hbar \left(\frac{1}{2mE_0} \right)^{1/2} \quad 51$$

can be calculated from the experimental E_0 values (Fig.1). Putting in (51) and (50) the available data:

$$\Delta_0 = E_0 = k_B \cdot 8.7K = 1.2 \cdot 10^{-15} \text{ erg};$$

the mass of atom: $m({}^4\text{He}) = 5.76 \cdot 10^{-24}g$ and $(m^*/m) = 0.16$, we obtain:

$$\lambda_0 \cong 14 \cdot 10^{-8}cm = 14\text{\AA} \quad 52$$

the corresponding most probable group velocity of ${}^4\text{He}$ atoms is: $v_{gr}^0 = 8.16 \cdot 10^3 cm/s$.

It is known from the experiment that the volume occupied by *one atom of liquid* ${}^4\text{He}$ is equal: $v_{({}^4\text{He})} = 46\text{\AA}^3/\text{atom}$. The edge length of the corresponding cubic volume is:

$$l = \left(v_{{}^4\text{He}} \right)^{1/3} = 3.58\text{\AA} \quad 53$$

From (52) and (53) we can calculate the number of ${}^4\text{He}$ atoms in the volume of primary librational (rotational) effecton at λ -point:

$$n_V^0 = \frac{V_{\text{ef}}}{V_{{}^4\text{He}}} = \frac{(9/4\pi)\lambda_0}{l^3} = 43 \text{ atoms} \quad 54$$

One edge of such an effecton contains $(43)^{1/3} \cong 3.5$ atoms of liquid ${}^4\text{He}$.

We must take into account, that these parameters can be *lower than the real* ones as in above simple calculations we did not consider the contribution of secondary effectons, transistons and deformons to total internal energy (see eq. 4.3).

On the other hand, in accordance with the mesoscopic model, the conditions of the maximum stability of primary effectons correspond to the *integer* number of particles in the edge of these effectons (see Chapter 6 and Fig. 7a of [1] or Fig.4a of [2]).

Consequently, we have to assume that the true number of ${}^4\text{He}$ atoms forming a primary effecton at λ -point is equal to $n_V^0 = 64$. It means that the edge of cube as the effecton shape approximation contains $q^0 = 4$ atoms of ${}^4\text{He}$:

$$n_e^0 = (n_V^0)^{1/3} = 64^{1/3} = 4 \quad 56$$

The primary librational effectons of such a type may correspond to rotons introduced by Landau to

explain the high heat capacity of He II.

The thermal impulses of ^4He atoms in these coherent clusters can totally compensate each other and the resulting impulse of primary effectons is equal to zero. Further decline in temperature gives rise to dimensions of primary effectons. The most stable of them contain in their ribs the integer number of helium atoms:

$$q = q^0 + n \quad 56a$$

where: $n = 1, 2, 3, \dots$

λ_0 , n_V^0 and n_e^0 can be calculated more accurately using eqs. (2.60) and (3.5) of [1], if the required experimental data on oscillatory spectroscopy and sound velocimetry are available.

5. Superfluidity as a hierarchic self-organization process

Let us consider now the consequence of the phenomena observed in ^4He in the course of temperature decline to explain Fig. 1 in the framework of mesoscopic model:

1. In accordance with our model lowering the temperature till the 4.2 K and gas-liquid first order phase transition occurs under condition (6.6). This condition means that the most probable wave B length of atoms related to their rotations or librations starts to exceed the average distance between ^4He atoms in a liquid phase:

$$\lambda = h/mv_{gr} \geq 3.58\text{\AA} \quad 57$$

The corresponding value of the most probable group velocity is

$$v_{gr} \leq 3.2 \cdot 10^4 \text{ cm/s.}$$

The translational thermal impulses of particles are usually bigger and waves B length smaller than those related to librations. In accordance with our model of first order phase transitions (Section 6.2 of [1]), this fact determines the difference in the temperatures of [gas \rightarrow liquid] and [liquid \rightarrow solid] transitions.

The freezing of liquid ^4He occurs at a sufficiently high pressure of ~ 25 atm only and means the emergency of primary translational effectons. A pressure increase as well as the drop in temperature declines the impulses between particles and stimulates distant Van der Waals interaction between them, responsible for coherent clusters formation.

In normal component of liquid ^4He II like in a usual liquid at $T > 0$ K, the existence of primary and secondary effectons, convertons, transions and deformons is possible. The contributions of each of these quasiparticles determine total internal energy (eq. 4.3 of [1, 2]), kinetic and potential energies (eqs. 4.33 and 4.36 of [1, 2]), viscosity (11.45), thermal conductivity (11.35), vapor pressure (11.26) of [1], paper [3] and many other parameters.

We assume that the lower branch in the excitation spectrum of Fig. 1 reflects the (a) state and the upper branch the (b) state of primary (lb and tr) effectons.

2. Decreasing the temperature to λ -point: $T_\lambda = 2.17\text{K}$ is accompanied by the condition (55), which stimulates Bose-condensation of atoms, increasing the dimensions of primary effectons as well as Bose-condensation of secondary effectons with nonzero resulting impulse. This leads to emergency of primary polyeffecton superfluid subsystem due to distant Van der Waals interactions and Josephson junctions between neighbouring effectons. It is accompanied by the (a)-states probability jump-way increasing ($P_a \rightarrow 1$, eq.4.10) and that of (b)-states decreasing ($P_b \rightarrow 0$, eq.4.11). Probability of primary and secondary deformons ($P_d = P_a \cdot P_b$; $\bar{P}_d = \bar{P}_a \cdot \bar{P}_b$) decreases correspondingly. In the excitation spectrum (Fig.1) these processes are displayed as the tendency of (b)-branch to (a)-branch due to degeneration of b-branch at very low temperature.

Like in the theory of 2nd order phase transitions proposed by Landau (Landau and Lifshits, 1976), we can introduce here the parameter of order as:

$$\eta = 1 - \kappa = 1 - \frac{P_a - P_b}{P_a + P_b} \quad 58$$

where: $\kappa = \frac{P_a - P_b}{P_a + P_b}$ is an equilibrium parameter.

One can see that at $P_a = P_b$, the equilibrium parameter $\kappa = 0$ and $\eta = 1$ (the system is far from 2nd order phase transition).

On the other hand, at conditions of phase transition: $T \rightarrow T_\lambda$ when $P_b \rightarrow 0$, $\kappa \rightarrow 1$ and parameter of order (η) tends to zero.

According to Landau's theory, the equality of his specific parameter of order to zero is a criterion of 2nd order phase transition. As usual, this transition is followed by a decrease in structural symmetry with a decline in temperature.

The important point of our scenario of superfluidity is the statement that the leftward shift of ($a \Leftrightarrow b$) equilibrium of the primary effectons (tr and lb) becomes stable starting from T_λ due to their polymerization "side by side". This process of Bose-condensation, including conversion of secondary effectons to primary ones, differs from condensation of an ideal Bose-gas described by eq. (1.26). Such kind of Bose-condensation means the enhancement of the concentration of (a)-state of primary effectons with lower energy, related to degeneration of the all others. The polymerization of primary effectons in He II gives rise to macroscopically long filament-like (or chain-like) polyeffectons. This process can be considered as self-organization on macroscopic scale. These filament-like polyeffectons, representing superfluid component, can form closed circles or three-dimensional (3D) isotropic networks in a vessel with He II.

The remnant fraction of liquid represent normal fraction of He II.

Polyeffectons are characterized by the dynamic equilibrium: [*assembly* \Leftrightarrow *deassembly*]. Temperature decrease and pressure increase have to shift this equilibrium to the left, increasing the surface of the primary effectons side-by-side interaction and number of Josephson junctions.

The probability of tunneling between coherent clusters increases also correspondingly.

The relative movement (sliding) of flexible "snake-like" polyeffectons occurs without phonons excitation in the volumes of IR deformons, equal to that of macrodeformons. Just macrodeformons excitation is responsible for dissipation and viscosity in normal liquids (see section 11.6).

The absence of macrodeformons excitation, related to polyeffectons emergency, explains the superfluidity phenomenon according to our model.

Breaking of symmetry in a three-dimensional polyeffecton network and its violation can be induced by external fields, like the gravitational gradient, mechanical perturbation and surface effects. It is possible as far the coherent polyeffecton system is highly cooperative and interaction between individual effectons as 3D standing waves is small.

In rotating cylindrical vessel, filament-like polyeffectons originate from 3D isotropic net and they tend to be oriented along the cylinder with their own rotation round their own axis in the direction opposite to that of cylinder rotation. In accordance with our model, this phenomenon represents the vortex filaments in He II discussed above. The radius of the filaments (42) is determined by the group velocity of the coherent ^4He atoms, which form part of the primary effectons ($v_{gr} = v_{sf}$). The numerical value of v_{gr} must be equal to or less than $6 \cdot 10^3 \text{ cm/s}$, this corresponding to conditions (55 and 56). At $T \rightarrow 0$, v_{gr} decreases and the filament radius (42) increases to reach the values corresponding to $v_{gr}^{\min} = v^0$ determined by the zero-point oscillations of ^4He atoms. Under these conditions the aggregation or polymerization of translational primary effectons in (a)-state can occur, leading to liquid-solid phase transition in ^4He .

The self-organization of highly cooperative coherent polyeffectons in λ -point and ($a \rightleftharpoons b$) equilibrium leftward shift should be accompanied by a heat capacity jump.

The mechanism, leading to stabilization of (a)- state of primary effectons as the first stage of their polymerization, is a formation of **coherent superclusters** from primary effectons without direct contacts. Stabilization of (a) states in **superclusters** could be resulted from macroscopic self-organization of matter in the volume of electromagnetic IR deformon due to distant Van der Waals interaction and *Vibro-gravitational interaction*, introduced in our theory between primary effectons (see section 10.4). These interactions between acoustic (a) states of the effectons are more effective, than between optic (b) states. They minimize the potential energy of the system and increase the probability of macroscopic Bose-condensation.

The successive mechanisms of super-clusterization and polymerization of primary effectons could be responsible for second order phase transitions, leading to emergency of superfluidity and superconductivity.

The second sound in such a model can be attributed to phase velocity in a system of polyeffectons or superclusters. The propagation of the second sound through chain polyeffectons or superclusters should be accompanied by their elastic deformation and [assembly \Leftrightarrow deassembly] equilibrium oscillations.

The third sound can be also related to the elastic deformation of polyeffectons and equilibrium constant oscillations of superclusters, but in the surface layer with properties different from those in bulk volume. In accordance with the mesoscopic theory, such a difference in surface and volume parameters is responsible (see eq.11.31-11.33) of [1] and [3] for surface tension (σ) in He II and its jump at λ -point. this increase in σ explain also the disappearance of cavitation bubbles at $T < T_\lambda$.

The fourth sound is the consequence of the increase in primary effecton dimensions and the change in their phase velocity as a result of He II interaction with narrow capillary's walls and thermal movement immobilization.

The normal component of He II is related to the fraction of He II atoms not involved in polyeffectons formation. This fraction composes individual primary and secondary effectons, maintaining the ability for ($a \Leftrightarrow b$) and ($\bar{a} \Leftrightarrow \bar{b}$) transitions. In accordance with our mesoscopic model, these transitions in composition of macroeffectons and macrodeformons are accompanied by the emission and absorption of heat phonons.

The manifestation of viscous properties in normal liquid and normal component of He II is related to fluctuations of concentration in the volume of macrodeformons (V_D^M) - see eqs. 11.48, 11.49 and 11.55 of [1].

On the other hand, macro- and superdeformons are absent in the superfluid component, as far in primary polyeffectons at $T < T_\lambda$: the probability of B-state of macroeffectons: $P_B = P_b \cdot \bar{P}_b \rightarrow 0$; the probability of A-state of the effectons: $P_A = P_a \cdot \bar{P}_a \rightarrow 1$ and, consequently, the probability of macrodeformons tends to zero: $P_D^M = P_B \cdot P_A \rightarrow 0$. Decreasing the probability of superdeformons $P_D^S = (P_D^M)_{tr} \cdot (P_D^M)_{lb} \rightarrow 0$ means the decreased concentration of cavitation bubbles and vapor pressure.

3. We can explain the decrease in $E(k)$ in Fig. 1 around $T = T_\lambda$ by reducing the contributions related to (b) of the primary effectons, degeneration of secondary effectons due to their Bose-condensation and concomitant elimination of the contribution of secondary acoustic deformons (i.e. phonons) to the total energy of liquid ^4He .

One can see from eqs. (11.45 - 11.53 of [1] and paper [3]) that under conditions of superfluidity at the absence of secondary effectons, when the life-time of secondary effectons and cycle-period of macroeffectons $(\tau^M)_{tr,lb}$ tends to zero, the viscosity also tends to zero: $\eta \rightarrow 0$.

In accordance with the mesoscopic theory of thermal conductivity (see eqs. 11.35 – 11.37), the elimination of secondary acoustic deformons at $T \leq T_\lambda$ must lead also to enhanced thermal conductivity. This effect was registered experimentally, indeed.

4. The increase in $E(k)$ in Fig. 1 at $T < T_\lambda$ can be induced by the enhanced contribution of primary polyeffectons to the total energy of He II and the factor: $U_{\text{tot}}/T_k = S^{-1}$ in the state equation (11.8 of [1]) and eq.(44).

The activity of the normal component of He II as a solvent for polyeffectons reduces and tends to zero at $T \rightarrow 0$. Under such condition ($T = 0$) superpolymerization and total Bose-condensation occur in ^4He .

The maximum in Fig. 1 at $0 < T < T_\lambda$ is the result of competition of two opposite factors: a rise in the total energy of He II due to progress of primary effectons polymerization and its reduction due to the decline in the most probable group velocity (v_{gr}), accompanied by secondary effectons and deformons degeneration (see eq.46). The latter process predominates at $T \rightarrow 0$. The development of a polyeffectons superfluid subsystem is accompanied by a corresponding diminution of the normal component in He II ($\rho_S \rightarrow 1$ and $\rho \rightarrow 0$). The normal component has a bigger internal energy than superfluid one.

The own dimensions of primary translational and librational effectons in composition of polyeffectons increases at $T \rightarrow 0$.

Verification of the inaccessibility of the b-state of primary effectons at $T \leq T_\lambda$

Let us analyze our formula (2.74 of [1], see also [2]) for the phase velocity of primary effectons in the (a)-state for the condition $T \leq T_\lambda$, when filament- like polyeffectons originate:

$$v_{ph}^a = \frac{v_S \frac{1-f_d}{f_a}}{1 + \frac{P_b}{P_a} \left(\frac{v_{res}^b}{v_{res}^a} \right)} \quad 59$$

where: v_S is the sound velocity; P_b and P_a are the thermoaccessibility of the (b) and (a) states of primary effectons; f_d and f_a are the probabilities of primary deformons and primary effectons in (a) state excitations (see eq. 2.66 of [1]).

One can see from (59), that if:

$$P_b \rightarrow 0, \text{ then } P_d = P_b \cdot P_a \rightarrow 0 \text{ and } f_d \rightarrow 0 \text{ at } T \leq T_\lambda$$

then phase velocity of the effecton in (a) state tends to sound velocity:

$$v_{ph}^a \rightarrow v_S \quad 60$$

For these λ – point conditions, the total energy of ^4He atoms forming polyeffectons due to Bose-condensation of secondary effectons (see 46) can be presented as:

$$E_{\text{tot}} \sim E_a = mv_{gr}v_{ph}^a \rightarrow mv_{gr}v_S \quad 61$$

where the empirical sound velocity in He II is $v_S = 2.4 \cdot 10^4 \text{ cm/s}$.

The kinetic energy of wave B at the same conditions is $T_k = mv_{gr}^2/2$. Dividing E_{tot} by T_k we have, using (47):

$$\frac{v_S}{v_{gr}} = \frac{E_{\text{tot}}}{2T_k} = \frac{1}{2S} = \frac{1}{2(m^*/m)} \quad 62$$

and

$$v_{gr}^0 = v_S \cdot 2S^0 = 2.4 \cdot 10^4 \cdot 0.32 = 7.6 \cdot 10^3 \text{ cm/s}. \quad 63$$

$m^* = 0.16m$ is the semiempirical effective mass at $T = T_\lambda$.

The most probable wave B length corresponding to (63) at λ -point:

$$\lambda^0 = h/mv_{gr}^0 = 15.1\text{\AA} \quad 64$$

The number of ^4He atoms in the volume of the same effecton calculated in accordance with (54) is equal: $q^0 = (n_v^0)^{1/3} = 3.8$.

This result is even closer to one predicted by the mesoscopic model (see 55) than (53). It

confirms that at $T \leq T_\lambda$ the probability of b-state $P_b \rightarrow 0$ and conditions (60) and (61) take place indeed.

In such a way our mesoscopic model of superfluidity explains the available experimental data on liquid ^4He in a noncontradiction manner **as a limit case of our mesoscopic viscosity theory for normal liquids.**

6. Superfluidity in ^3He

The scenario of superfluidity, described above for Bose-liquid of ^4He ($S = 0$) in principle is valid for Fermi-liquid of ^3He ($S = \pm 1/2$) as well. A basic difference is determined by an additional preliminary stage related to the formation of Cooper pairs of ^3He atoms with total spins, equal to 1, i.e. with boson's properties. The bosons only can form effectons as coherent clusters containing particles with *equal* energies.

We assume in our model that Cooper's pairs can be formed between neighboring ^3He atoms. It means that the minimum number of ^3He atoms forming part of the primary effecton's edge at λ -point must be 8, i.e. two times more than that in ^4He (condition 55). Correspondingly, the number of ^3He atoms in the volume of an effecton is $(n_V^0)_{^3\text{He}} = 8^3 = 312$. These conditions explains the fact that superfluidity in ^3He arises at temperature $T = 2.6 \cdot 10^{-3}\text{K}$, i.e. lower than that in ^4He . The formation of flexible filament-like polyeffectons, representing macroscopic Bose-condensate in liquid ^3He responsible for superfluidity, is a process, similar to that in ^4He described above.

7. Superconductivity

General properties of metals and semiconductors

The dynamics of conductance electrons in metals and semiconductors is determined by three main factors (Kittel, 1978, Ashcroft and Mermin, 1976, Blakemore, 1985):

- 1. The electric field influencing the energy of electrons.**
- 2. The magnetic fields changing the direction of electrons motions.**
- 3. Scattering on the other electrons, ions, phonons, defects.**

The latter factor determines the values of the electron conductance and resistance.

In spite of the small mean distances between electrons in metals (2-3) \AA their mean free run length at room temperatures exceeds 10^4\AA and grows by several orders at $T \rightarrow 0$. It is related to the fact that **only electrons having energy higher than Fermi energy (ϵ_F) may be involved in collisions**. The fraction of these electrons in the total number of electrons is very small and decreases on lowering the temperature as $(kT/\epsilon_F)^2$. At room temperatures the scattering of electrons in metals occurs mainly on phonons.

The mean free run length of electrons in indium at 2K is about 30 cm.

The analysis of electric and magnetic fields influence on an electron needs the notion of its effective mass (m^*). It is introduced as a proportionality coefficient between the force acting on the electron and the acceleration (a) in the electric field (E):

$$F = -eE = m^*a; \quad a = dv_{gr}/dt \quad 65$$

In a simple case of an isotropic solid body the effective mass of an electron is a scalar (Kittel, 1978):

$$m^* = \frac{\hbar^2}{d^2\epsilon/dk^2} \quad 66$$

where ϵ is the kinetic energy of an electron, having a quadratic dependence on the wave number ($k = 1/L_B$):

$$\epsilon = \frac{\hbar^2 k^2}{m^*} = \frac{\hbar^2}{2mL^2} \quad 67$$

In a general case, for electrons in solid bodies with a complex periodic structure, the effective mass is a tensor:

$$[m_{ij}^*] = \hbar^2 / [\partial^2 \epsilon / \partial k_i \partial k_j] \quad 68$$

The effective mass tensor can have positive components for some directions and negative for others.

Plasma oscillations

At every displacement of the electron gas relative to the subsystem of ions in a solid body, a returning electric field appears. As a consequence of that, the subsystem of electrons will oscillate relative to the subsystem of ions with the characteristic plasma frequency (Ashcroft and Mermin, 1976):

$$\omega_{pl} = 2\pi\nu_{pl} = \left(\frac{4\pi n e^2}{m^*} \right)^{1/2} \quad 69$$

where: n is the number of electrons in 1cm^3 , e is the charge and (m^*) is the effective mass of an electron.

The quantified collective oscillations of electron plasma are termed *plasmons*. With decreasing n from 10^{22} to 10^{10}cm^{-3} the frequencies ω_{pl} decrease from $6 \cdot 10^{15}\text{s}^{-1}$ to $6 \cdot 10^3\text{s}^{-1}$. For metals ω_{pl} corresponds to an ultraviolet frequency range, and for semiconductors - to an IR frequency range.

For longitudinal plasma oscillations at small wave vectors the dependence of frequency on the wave number ($k = 1/L = 2\pi/\lambda$) can be approximately represented as (Kittel, 1978):

$$\omega \approx \omega_{pl} \cdot \left(1 + \frac{3k^2 v_F^2}{10\omega_{pl}^2} + \dots \right) \quad 70$$

where: v_F is the Fermi velocity of an electron (see eq.77).

The screening length (l), characterizing the electron-electron interaction in plasmon when Fermi-gas is degenerated is equal to:

$$l = v_F/v_p \sim 1\text{\AA} \text{ in metals} \quad 71$$

For the cases of non-degenerated Fermi-gas, when the concentrations of free electrons are sufficiently low (in semiconductors) or at high temperatures $T \sim 10^4\text{K}$, the screening length (l_d) is dependent on thermal electron velocity:

$$v_{th} = \left(3k_B T/m^* \right)^{1/2} \quad 72$$

and

$$l_D = v_{th}/v_p \cong \left(\frac{\epsilon k_b T}{4\pi n e^2} \right)^{1/2} \quad 73$$

where: v_{pl} corresponds to (69), ϵ is the dielectric constant.

For example, if in a semiconductor $n = 5 \cdot 10^{17}\text{cm}^{-3}$ and $\epsilon = 12$, then $l_D = 60\text{\AA}$ (March, Parrinello, 1982).

Fermi energy

The notion of Fermi energy (ϵ_F) can be derived from the Pauli principle forbidding the fermions to be in the same energetic states.

The formula for Fermi energy for the case of ideal electron gas includes the electron mass (m), the Plank constant ($h = 2\pi\hbar$) and the concentration of free electrons ($n_e = N_e/V$):

$$\epsilon_F = \frac{\hbar^2}{2m} \left(\frac{3}{8}\pi n_e \right)^{3/2} = \frac{2\pi^2 \hbar^2}{m} \left(\frac{3}{8}\pi n_e \right)^{3/2} \quad 73a$$

where N_e is the number of free electrons in selected volume (V). For a real electron gas, m must be substituted by its effective mass: $m \rightarrow m^*$.

The formula (73a) can also be derived using the idea of standing waves B of the unbind electrons of matter. The condition under which the concentration of twice polarized standing waves B of electrons is equal to the concentration of electrons themselves:

$$n_B^F = \frac{N_e}{V} = \frac{8\pi}{3(\lambda_B^F)^3} \quad 74$$

The wave B length of an electron corresponding to this condition is:

$$\lambda_B^F = \left(\frac{8\pi V}{3N_e} \right)^{1/3} = \frac{\hbar}{mv_{gr}^F} \quad 75$$

The kinetic energy of the unbind electrons waves B (T_k) could be expressed through their length and mass. It appears that the kinetic energy of the electrons standing waves B, limited by their concentration is equal to Fermi energy:

$$T_k^F = \frac{\hbar^2}{2m\lambda_B^F} = \frac{\hbar^2}{2m} \left(\frac{3n_e}{8\pi} \right)^{2/3} = \frac{P_F^2}{2m} = \epsilon_F, \quad 76$$

where Fermi impulse:

$$P_F = mv_F = \hbar \left(\frac{3n_e}{8\pi} \right)^{1/3} = \hbar(3\pi^2 n_e)^{1/3} \quad 77$$

The Fermi energy corresponds to Fermi temperature (T_F):

$$\epsilon_F = kT_F = \hbar v_F \quad 78$$

At $T < T_F$ electron gas is in a strongly "compressed" state. The more the relation $(T/T_F) = kT/\epsilon_F$, the more the probability of the appearance of "free volume" in a dense electron gas. On lowering the temperature, when the impulse of electrons decreases and the heat wave B length increases, the "effective pressure" of the electron gas grows, leading to its Bose-condensation.

Cyclotronic resonance

The magnetic field B_z in the direction (z) influencing the electron by the Lorentz force, changes the direction of its motion without changing the energy. If an electron's energy does not dissipate, then the electrons rotate in the plane xy, around z-axis. Such an electron with the effective mass m^* has a circulation orbit of the radius r, with rotation frequency ω_c . From the condition of equality between the Lorentz force ($r\omega_c eB_z$) and the centrifugal force ($m^*\omega_c^2 r$) the formula is derived for angular cyclotron frequency (Kittel, 1978):

$$\omega_c = eB_z/m^* \quad 79$$

The kinetic energy, corresponding to the rotation is equal to:

$$T_k = \frac{1}{2} m^* (\omega_c)^2 r^2 \quad 80$$

In the range of radio-frequencies (ω) such a value of the magnetic induction B_z can be selected that at this value the resonance energy absorption occurs, when $\omega = \omega_c$.

Such experiments on the cyclotron resonance can be done to determine m^* in selected directions.

In a simple case, an electron revolves around the Fermi sphere with the *zero impulse*

component in z-direction.

The radius of this sphere is determined by the Fermi impulse P_F (see eq. 77). In the real space:

$$r_F \sim \hbar/P_F \quad 81$$

The energy of free particles near the Fermi surface:

$$\epsilon(P_F) = v_F(P - P_F) \quad 82$$

where: v_F and $P_F = m^* v_F$ are the Fermi velocity and impulse: $P > P_F$ is the impulse of thermal electron at $T > 0$ near the Fermi surface.

The solution of the Schrödinger equation, modified by Landau for electrons in a magnetic field in real space leads to the following total energy eigenvalues (Blakemore, 1985):

$$\epsilon = \frac{\hbar^2 k_z^2}{m^*} + \left(l + \frac{1}{2}\right) \hbar \omega_c, \quad 83$$

where the first term of the right part represent the energy of *translational* motion of electrons, which does not depend on magnetic field magnitude; $k_z = 1/L_z$ is the wave number of this motion; the second term is responsible for rotational energy, $l = 0, 1, 2, \dots$ is the integer quantum number for rotational motion in magnetic field B_z . Every value of l means a corresponding Landau level.

Thus, free electrons in a magnetic field move along the helical trajectory of the radius:

$$r_l = [(2l + 1)\hbar/m^* \omega_c]^{1/2} \quad 84$$

At the transition from real space to the wave number space, the radius of the orbit (k_p) and its area is quantified as:

$$S_l = \pi k_p^2 = \frac{2\pi e B}{\hbar c} \left(l + \frac{1}{2}\right) \quad 85$$

This formula is valid not only for the free electron model, but also for real metals.

The magnitude $2\pi(\hbar c/e)$ termed a flux quantum.

In a strong magnetic field the quantization of electrons energy leads to the periodic dependence of the metal magnetic moment on the magnetic field (B): the de Haas - van Alfen effect (Kittel, 1978, Ashcroft and Mermin, 1976).

Electroconductivity

According to the Sommerfeld theory (Blakemore, 1985), electroconductivity (σ) depends on the free run time of an electron (τ) between collisions:

$$\sigma = ne^2\tau/m, \quad 86$$

where: n is the concentration of electrons, (e) and (m) are electron charge and mass.

The free run time is equal to the ratio of the average free run distance (λ) of electrons to the Fermi speed (v_F):

$$\tau = \lambda/v_F \quad 87$$

The free run distance is determined by scattering at defects (λ_D) and scattering at phonons (λ_{ph}):

$$1/\lambda = 1/\lambda_D + 1/\lambda_{ph} \quad 88$$

The resistance ($R = 1/\sigma$) could be expressed as:

$$R = 1/\sigma_D + 1/\sigma_{ph} = R_D + R_{ph} \quad 89$$

the contribution R_D depends mainly on the concentration of the conductors defects, and the phonon contribution R_{ph} depends on temperature.

Formula (89) expresses the Mattisen rule.

A transition to a superconducting state means that the free run time and distance tend to infinity: $\tau \rightarrow \infty$; $\lambda \simeq \lambda_{ef} = \hbar/P_{ef} \rightarrow \infty$, while the *resulting* group velocity of the electrons (v_{gr}^{res}) and

impulses tends to zero:

$$P_{ef} = m \cdot v_{gr}^{res} \rightarrow 0$$

The emergency of macroscopic Bose-condensation of secondary ionic effectons and Cooper pairs corresponds to this condition.

We assume in our hierarchic model, that the absence of the non-elastic scattering and dissipation of electrons energy is observed as superconductivity, when the probability of secondary ionic effectons and deformons tends to zero, leading to emergency of primary electronic polyeffectons.

Let us consider first a conventional microscopic approach to the problem of superconductivity.

8. Microscopic theory of superconductivity (BCS)

This theory (BCS) was created by Bardin, Cooper and Schriffer in 1957. The basic, experimentally proven assumption of this theory, is that electrons at sufficiently low temperatures are grouped into Cooper pairs with oppositely directed spins - Bose-particles with a zero spin. The charge of the pair is equal to $e^* = 2e$ and mass $m^* = 2m_e$.

Such electron pairs obey the Bose-Einstein statistic. The Bose- condensation of this system at the temperature below the Bose-gas condensation temperature ($T < T_k$) leads to the superfluidity of the electron liquid. This superfluidity (analogous to the superfluidity of liquid helium) is manifested as superconductivity.

According to BCS's theory, the Cooper electron pair formation mechanism is the consequence of virtual phonon exchange through the lattice.

The energy of binding between the electrons in a pair is very low: $2\Delta \sim 3kT_c$. It determines a minimum energetic gap (Δ) separating a state of superconductivity from a state of usual conductivity.

Notwithstanding that the kinetic energy of electrons in a superconducting state is greater than ϵ_F , the contribution of the potential energy of attraction between electron pairs is such that the total energy of the superconducting state (E_a^e) is smaller than the Fermi energy (ϵ_F) (Kittel, 1978). The presence of the energetic gap Δ makes a superconducting state stable after switching-off external voltage. The middle of the gap coincides with the Fermi level.

The rupture of a pair can happen due to photon absorption by superconductor with the energy: $\Delta = h\nu_p \approx 3kT_c$. Superconductivity usually disappears in the frequency range $10^9 < \nu_p < 10^{14} s^{-1}$.

In the BCS theory, the magnitude Δ is proportional to the number of Cooper pairs and grows on lowering the temperature.

The excitation energy of quasiparticles in a superconducting state, which is characterized by the wave number (k), is:

$$E_k = (\epsilon_k^2 + \Delta^2)^{1/2}, \quad 90$$

where

$$\epsilon_k = \frac{\hbar^2}{m}(k^2 - k_F^2) \approx \frac{\hbar^2}{m}k_F(k - k_F) \quad 91$$

and

$$\delta k_F = (k - k_F) \ll k_F = 1/L_F$$

The critical speed of the electron gas (v_c), for exciting a transition from a superconducting state to a normal one is determined from the condition:

$$E_k = \hbar k v_c \text{ and } v_c = \frac{E_k}{\hbar k} \quad 92$$

The wave function $\Phi(r)$, which describes the properties of electron pairs in the BCS theory, is the superposition of one-electron functions with energies in a range of about 2Δ near ϵ_F . Therefore, the dispersion of impulses for one-electron levels involved in the formation of pairs is expressed as:

$$\Delta = \delta\epsilon_F = \delta\left(\frac{P_F^2}{2m}\right) = \left(\frac{P_F}{m}\right)\delta P_F \approx v_F \delta P_F \quad 93$$

The characteristic coherence length (ξ_c) of the pair function $\Phi(r)$ has the value (Ashcroft and Mermin, 1976, Lifshits and Pitaevsky, 1978):

$$\xi_c \sim \hbar/\delta P_F \simeq \frac{\hbar v_F}{\Delta} \simeq \frac{1}{k_F} \frac{\epsilon_F}{\Delta} \quad 94$$

The magnitude (ϵ_F/Δ) is usually $10^3 - 10^4$, and $k_F = 1/L_F \sim 10^8 \text{ cm}^{-1}$. Thus, from (94):

$$\xi_c \sim (10^3 - 10^4) \text{ \AA} \quad 95$$

Inside the region of coherence length (ξ_c) there are millions of pairs. The impulses of pairs in such regions are correlated in such a way that their resulting impulse is equal to zero.

At $T > 0$ some of the pairs turn to a dissociated state and the concentration of superconducting electrons (n_s) decreases. The coherence length (ξ_c) also tends to zero with increase in temperature.

The important parameter, characterizing the properties of a superconductor is the value of the critical magnetic field (H_c), above which the superconductor switches to a normal state.

With a rise in the temperature of the superconductor when $T \rightarrow T_c$, the critical field tends at zero: $H_c \rightarrow 0$. And vice versa, at lowering of temperature, when

$T < T_c$, the H_c grows up as:

$$H_c = H_0[1 - (T/T_c)^2]$$

where H_0 corresponds to $T = 0$.

The Meisner effect - the "forcing" of the outer magnetic field out of the superconductor is also an important feature of superconductivity.

The depth of magnetic field penetration into the superconductor (λ) (Kittel, 1978, Ashcroft and Mermin, 1976) is:

$$\lambda = (m\epsilon_0 c^2/e^2 n_s)^{1/2} \simeq (10^{-6} - 10^{-5}) \text{ cm},$$

where n_s , the density of electrons in a superfluid state; ϵ_0 - dielectric constant.

On temperature raising from 0 K to T_c , the λ grows as:

$$\lambda = \frac{\lambda_0}{\left[1 - \left(\frac{T}{T_c}\right)^4\right]^{1/2}}, \quad 96$$

where: λ_0 corresponds to λ at $T = 0$.

The superconductors with magnetic field penetration depth (λ) less than coherence length ξ ::

$$\lambda \ll \xi \quad 97$$

are termed first order superconductors and those with

$$\lambda \gg \xi \quad 98$$

are second order ones.

Nowadays, in connection with the discovery of high temperature superconductivity (Bednorz, Muller, 1986, Nelson, 1987) the mechanism of stabilizing electron pairs by means of virtual phonons in the BCS theory evokes doubts.

9. Mesoscopic scenario of superconductivity

We propose a new mechanism of electron pair formation and their subsequent Bose-condensation, without virtual phonons as mediators. Such a process is analogous to the formation of primary polyeffectons in liquid helium related to superfluidity phenomena (see Section 5).

Two basic questions must be answered in relation to the emergence of superconductivity:

I. Why does energy dissipation in the system [conductivity electrons + lattice] disappear at $T \leq T_c$?

II. How does the coherence in this system, related to electron pair formation, originate ?

It will be shown below how these problems can be solved in the framework of our Hierarchic (mesoscopic) theory.

The following factors can affect electron's dynamics and scattering near Fermi energy:

1. Interaction of electrons with primary and secondary ionic effectons in acoustic (a) and () states, stimulating Cooper pairs formation;
2. Interaction with primary and secondary effectons of lattice in acoustic (b) and (\bar{b}) states, leading to origination of polarons and Cooper pairs dissociation;
3. Interaction with transistons in the course of ($a \leftrightarrow b$) and ($\leftrightarrow \bar{b}$) transitions of primary and secondary effectons;
4. Interaction with [tr/lb] convertons (interconversions between primary translational and librational effectons);
5. Interaction with primary electromagnetic deformons;
6. Interaction with secondary acoustic deformons (possibility of polaron formation);
7. Interaction with macroeffectons in A- and B-states;
8. Interaction with macro- and superdeformons (possible emergence of defectons).

It follows from our model that the oscillations of all types of quasiparticles in conductors and semiconductors are accompanied not only by electron-phonon scattering, but also by electromagnetic interaction of primary deformons with unbind electrons.

At $T > T_c$ the fluctuations of unbind electrons with energy higher than Fermi one under the influence of factors (1 - 8) are random (noise-like) and no selected order of fluctuations in normal conductors exists. It means that ideal Fermi-gas approximation for such electrons is sufficiently good. In this case, the effective electron mass can be close to that of a free electron ($m^* \simeq m$).

Electric current in normal conductors at external voltage should dissipate due to fluctuations and energy exchange of the electrons with lattice determined by factors (1 - 8).

Coherent in-phase acoustic oscillation of the ionic primary effectons in (a)-state is the "ordering factor" simulating electron gas coherence due to electromagnetic interactions. But its contribution in normal conductors at $T > T_c$ is very small. For an ideal electron gas, the total energy (E_{tot}) of each electron as wave B is equal to its kinetic energy (T_k), as far potential energy ($V = 0$):

$$E_{tot} = \hbar\omega = \frac{\hbar^2}{2m\Lambda^2} = T_k + V = \frac{\hbar^2}{2mL^2} \quad 99$$

where:

$$T_k = \frac{\hbar^2}{2mL^2} \quad 100$$

and

$$L = \frac{\hbar}{m v_{gr}} = \frac{1}{k} \quad 101$$

One can see from (100) that for an ideal gas, when $m = m^*$, the most probable amplitude (A) and wave B length (L) are equal:

$$A = L, \text{ if } E_{tot} = T_k \text{ and } V = 0 \quad 102$$

Like in case of liquid helium at conditions of superfluidity, Bose- condensation in metals and semiconductors is related to an increase in the concentration of the (a)-state of primary effectons with the lowest energy and a corresponding decrease in the concentrations of all other excitations. The Bose-condensation and degeneration of secondary ionic effectons and deforms, followed by formation of electronic polyeffectons from Cooper pairs is responsible for second order phase transition like superconductivity.

The cooperative character of 2nd order phase transition [conductor \rightarrow superconductor] is determined by a feedback reaction between the lattice and electron subsystems. It means that the collective Bose-condensation in both subsystems is starting at the same temperature: $T = T_c$.

Under such conditions the probabilities of the (a)-states of ionic (P_a^i) and electronic (P_a^e) effectons tend to 1 at $T \leq T_c$:

$$\left. \begin{array}{l} P_a^i \rightarrow 1; P_a^e \rightarrow 1 \\ P_b^i \rightarrow 0; P_b^e \rightarrow 0 \end{array} \right\} \quad 103$$

The equilibrium parameter for both subsystems:

$$\kappa^{i,e} = \left(\frac{P_a - P_b}{P_a + P_b} \right)^{i,e} \rightarrow 1 \quad 104$$

and the *order parameter*:

$$\eta^{i,e} = (1 - \kappa^{i,e}) \rightarrow 0 \quad 105$$

like in a 2nd order phase transition for liquid helium (see eq. 58).

In our model the coherent Cooper pairs are formed as Bose particles with resulting spin equal to 0 and 1 from neighboring electrons, in contrast to the BCS theory, which assumes phonons - mediated interaction between distant electrons.

Such pairs can compose primary electron's effectons (e-effectons) as a coherent cluster with a resulting impulse *equal to zero*. Formation of secondary e-effectons with nonzero resulting impulse is possible also. Just the interaction of this secondary e-effectons with lattice is responsible for electric resistance in normal conductors. Degeneration of such type of excitations in the process of their Bose-condensation and their conversion to primary e-effectons means the emergency of superconductivity.

The in-phase coherent oscillations of the integer number of electron pairs forming primary e-effectons correspond to its acoustic (a)- state, and the counterphase oscillations to its optic (b) state, like in ionic or molecular effectons.

We assume that superconductivity can originate only when the fraction of unbind coherent electrons forming *primary e-effectons in definite regions of conductor strongly prevails over the fraction of noncoherent secondary e-effectons. Due to feedback reaction between subsystems of lattice and electrons this fraction should be equal to ratio of wave length of primary and secondary ionic effectons. This condition can be introduced as:*

$$(II) : \left(\frac{\lambda_a^e}{\bar{\lambda}_a^e} \right)_{T_c} = \left(\frac{v_s/\lambda_a}{v_s/\bar{\lambda}_a} \right)_{T_c} \geq 10 \quad 106$$

where: $\lambda_a^e / \bar{\lambda}_a^e$ is the ratio of wave lengths of primary and secondary e- effectons, equal to that of primary and secondary effectons of lattice; sound velocity is equal to lattice primary effectons phase velocity in (a) state: $v_s \simeq v_{ph}^a$ under conditions corresponding to

(60).

$$\lambda_a^e = \lambda_a^i \simeq 10 \cdot \bar{\lambda}_a^e \simeq 20 \cdot n_c^{1/3} \quad 107$$

are the periods of electron's oscillations modulated by the oscillations of primary and secondary ionic effecton's in the acoustic states.

As far the oscillations of e-pairs in the Bose-condensate (a-state of e- effectons) are modulated by the electromagnetic field, radiated by oscillating ions in the (a)-state of primary ionic effectons they must have the same frequency.

Condition (106) means that the number of electrons in the volume ($V_e \sim \lambda^3$) of primary e-effectons is about 10^3 times more than that in secondary e-effectons. The lattice and electronic effectons subsystems are spatially compatible.

As far the effective mass (m^*) of the electrons in the coherent macroscopic Bose-condensate organized by e-polyeffectons at conditions of superconductivity tends to infinity:

$$\begin{aligned} T &< T_c \\ m^* &\mapsto \infty \\ T &\rightarrow 0 \end{aligned} \quad 108$$

the plasma frequency (eq. 69) tends to zero:

$$\begin{aligned} T &< T_c \\ 2\pi\nu_{pl} = \omega_{pl} &\mapsto 0 \\ &\text{at } T \rightarrow 0 \end{aligned}$$

and, consequently, the screening length (eq.71) tends to infinity: $l \rightarrow \infty$. This condition also corresponds to that of macroscopic Bose-condensation emergency

Under these conditions an impulse (see eq.93) originates in addition to Fermi's one:

$$\delta P_F = \hbar \delta k_F > 0 \quad 109$$

but a decrease in the potential energy of both electron's and ion's subsystems due to leftward ($a \leftrightarrow b$) equilibrium shift leads to the emergence of the gap near the Fermi surface (2Δ) depending on the difference of energy between (a) and (b) states of primary effectons.

The linear dimension of coherent primary electronic effectons (e-effectons), which is equal to coherence length in the BCS theory (see eq. 94) is determined by additional impulse δP_F :

$$\xi = \lambda_a^e = \frac{v_s}{v_a} = h/\Delta p_F = h/m_e \Delta v_F \quad 110$$

In turn, the primary e-effecton in (a)-state can form e- polyeffectons as a result of their polymerization. The starting point of this collective process represents macroscopic Bose - condensation and second order phase transition in accordance with our model.

The energy gap between normal and superconductive states can be calculated directly from our mesoscopic theory, as the difference between the total energy of matter before $\left[U_{\text{tot}}^{T>T_c} \right]$ and after $\left[U_{\text{tot}}^{T<T_c} \right]$ the second order phase transition:

$$2\Delta = U^{T>T_c} - U^{T<T_c} \quad 111$$

However, such experimental parameters as sound velocity, density and the positions of bands in a far IR region must be available around transition temperature (T_c) for calculation of (111).

This gap must be close to the energy of $(a \rightarrow b)^i$ transitions of ionic primary effectons, related to the energy of $(a \rightarrow b)^e$ transitions of the electronic e-effectons (see 103 - 105).

This statement of our superconductivity model coincides well with the experimental destruction of superconductivity state by IR-radiation with minimum frequency (ν_g),

corresponding to the energy gap (2Δ) at given temperature:

$$hv_g = 2\Delta \sim (E_b - E_a) \quad 112$$

Another general feature of superconductivity for low- and high-temperature superconductors is the almost constant ratio:

$$\frac{2\Delta_0}{k_B T_c} \simeq 3.5 \quad 113$$

where the gap: $\Delta = \Delta_0$ at $T = T_c$ and $\Delta = 0$ at $T > T_c$.

It will be shown below that the experimental result (113) is related to condition (106) of our mesoscopic model of superconductivity.

Considering (112), (113) and (2.27), the frequency of a primary ionic effectons in a-state near transition temperature is:

$$v_a^i = \frac{v_g^0}{\exp\left(\frac{2\Delta}{kT_c}\right) - 1} \simeq v_g^0/32.1 = 0.03(2\Delta/h) \quad 114$$

consequently:

$$hv_a^i = 0.03 \cdot 2\Delta \quad 115$$

The frequency of secondary lattice effectons in (\bar{a}) –state in accordance with (2.54) is:

$$\bar{v}_a^i = \frac{v_a^i}{\exp\left(\frac{hv_a^i}{kT_c}\right) - 1}$$

as far:

$$\frac{hv_a^i}{kT_c} = 0.03 \frac{2\Delta}{kT_c} = 0.1 \ll 1 \quad 116$$

$$\text{we have: } \left[\exp\left(\frac{hv_a^i}{kT_c}\right) - 1 \right]^{-1} \sim \frac{kT_c}{hv_a^i} \sim 10$$

consequently:

$$v_a \simeq kT_c/h, \quad 117$$

Now, using (17), (116) and (113) we confirm the correctness of condition (106):

$$\lambda_a^e/\bar{\lambda}_a^e = (\bar{v}_a/v_a)^i = \left[\exp\left(\frac{hv_a^i}{kT_c}\right) - 1 \right]^{-1} \sim 10 \quad 118$$

where:

$$\lambda_a^e = h/2m_e(v_{gr}^a)^e = (v_s/v_a)^i \quad 119$$

is the most probable wave B length of coherent electron pairs composing a primary e-effecton; $(v_{gr}^a)^e$ is a group velocity of electron pairs in a-state of primary e-effectons, stimulated by ionic lattice oscillations:

$$\bar{\lambda}_a^e = h/2m^* \bar{v}_{gr}^a = (v_s/\bar{v}_a)^i \quad 120$$

is the mean wave B length of electron pair forming the effective secondary e-effecton.

Our theory predicts also the another condition of coherency between ionic and electronic subsystems, leading to superconductivity, when the linear dimension of primary translational ionic effectons grows up to the value of coherence length (see eq.94):

$$(\lambda_a^i)_{tr} = \frac{v_s}{(v_a^i)_{tr}} \geq \zeta = \frac{\hbar v_F}{\Delta} \quad 121$$

In simple metals a relation between sound (v_s) and Fermi velocities (v_F) is determined by the electron to ion mass ratio $(m_e/M)^{1/2}$ (March and Parinello, 1982):

$$v_s = \left(\frac{zm_e}{2M} \right)^{1/2} \cdot v_F, \quad 122$$

where: z is the valence of ions in a metal.

Putting $(v_a^i)_{tr}$ from eq.(115) and eq.(122) in condition (121), and introducing instead electron mass its effective mass (m^*) in composition of e-effecton , we obtain at $T = T_c$:

$$m^* \stackrel{T_c}{\simeq} 2 \cdot 10^{-4} \cdot \frac{M}{z} \quad 123$$

As far we assume here that at transition temperature (T_c) the volumes of primary lattice (ionic) effectons and primary e-effectons coincide, then the number of electrons in the volume of primary effectons is:

$$N_e = n_e \cdot V_{ef}^i = n_e \cdot \frac{9}{4\pi} \left(\frac{v_s}{v_a^i} \right)_{T_c}^3 \quad 124$$

where: n_e is a concentration of the electrons; $V_{ef}^i = V_{ef}^e$ is the volume of primary translational ionic effectons.

Using the relation between primary and secondary ionic waves B (106) at T_c as $\lambda_a = 10\bar{\lambda}_a$, taking into account (117) and (121) we got:

$$\left(\frac{zm_e^*}{2} \right)^{1/2} \cdot \frac{1}{T_c M^{1/2}} = \frac{k}{10 \cdot 2\pi\Delta} \quad 125$$

or:

$$T_c(M/z)^{1/2} = \frac{10\pi\Delta}{k} \cdot (2m_e^*)^{1/2} \quad 126$$

If for the different isotopes the energetic gap is constant ($2\Delta \simeq const$), then the left part of (126) is constant also. Such an important correlation between transition temperature (T_c) and isotope mass (M) is experimentally confirmed for many metals:

$$T_c \simeq \left(\frac{h}{k_B} \right) \cdot \frac{v_s}{2n_e^{1/3}} \quad 127$$

This result as well as (117) can be considered as evidence in proof for our model of superconductivity.

It leads from (127) that the more rigid is lattice and the bigger is sound velocity (v_s), the higher is transition temperature. Anisotropy of v_s means the anisotropy of superconductor properties and can be affected by external factors such as pressure.

It was shown in this article, that all most important phenomena, related to turbulence, superfluidity and superconductivity can be explained in the framework of our Hierarchic theory.

REFERENCES

- Ashcroft N., Mermin N. Solid state physics. N.Y.:Helt, Rinehart and Winston, 1976.
- Bardeen J., Cooper L.N., Schrieffer J.R. Phys. Rev., 108, 1175, 1957
- Bardeen J., Schrieffer J.R. Progr. Low. Temp. Phys, 3, 170, 1960.
- Beizer A. Basic ideas of modern physics. Nauka, Moscow, 1973.
- Bednorz J.G., Muller K.A. Z.Phys.B. Condensed Matter, 64, 189, 1986
- Blakemore J.S. Solid state physics. Cambridge University Press, Cambridge, N.Y. e.a, 1985.
- Bogolyubov N.N. Lectures on quantum statistics. Collected works. Vol.2. Kiev, 1970.
- Cooper L.N. Phys. Rev., 104, 1189.
- Feynman R. Statistical mechanics.
- Feynman R. The character of physical law. Cox and Wyman Ltd., London, 1965.
- Frolich H. Phys. Rev., 79, 845, 1950.
- Käiväräinen A.I. Theory of condensed state as a hierarchical system of quasiparticles formed by phonons and three-dimensional de Broglie waves of molecules. Application of theory to thermodynamics of water and ice. J.Mol.Liq. 1989a, 41, 53 – 60.
- Käiväräinen A.I. Mesoscopic theory of matter and its interaction with light. Principles of self-organization in ice, water and biosystems. University of Turku, Finland 1992, pp.275.
- Kittel Ch. Thermal physics. John Wiley and Sons, Inc., N.Y., 1975.
- Kittel Ch. Introduction to the solid state physics. Nauka, Moscow, 1978 (in Russian).
- London F. On the Bose-Einstein condensation. Phys.Rev. 1938, 54, 947.
- London F. Superfluids, v1, Wiley, 1950
- Landau L.D., Lifshits E.M. Statistical physics. Nauka, Moscow, 1976 (in Russian).
- Prokhorov A. M. (Ed.) Physical encyclopedia. Vol.1-4. Moscow, 1988.
- Schrieffer J. 1957
- Zeldovitch Ya.B., Khlopov M.Yu. Drama of concepts in cognition of nature. Nauka, Moscow, 1988.